Power grids as complex networks

Author: Mikk Kruusalu
Supervisor: Marco Patriarca

Tallinn 2025

1 Introduction

Electricity is a resource taken for granted in the modern world. With the increasing demand more load relies on the power grid and it needs to scale. On the other hand, since much of the modern cities and countries core functions rely on electricity, it is also something that needs to be very reliable and fault tolerant in cases of attacks or random failures. The latter, is what fuels much of the research in the power grid network theory [1].

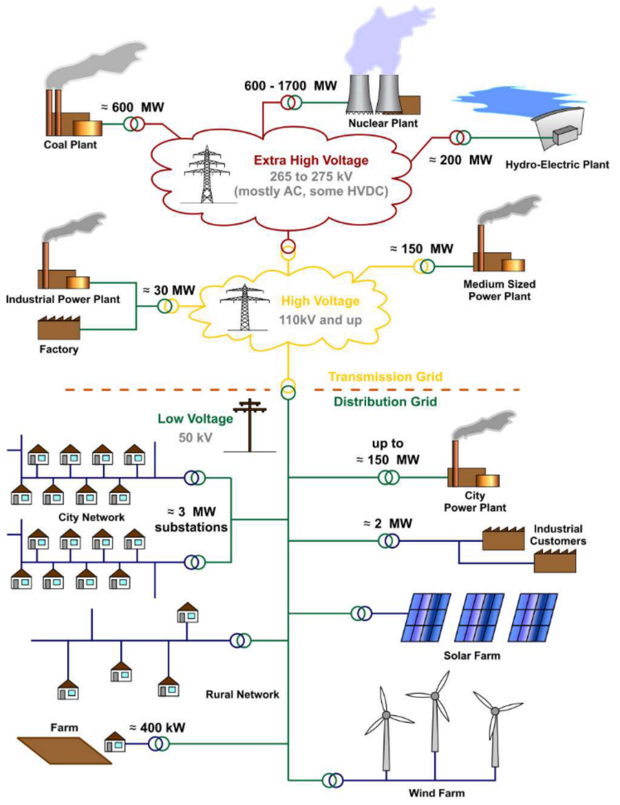

Figure 1: Typical power grid topology. Figure is taken from [1]

The main components of a power grid are

- energy production plants,
- transmission grid that transmits energy to every region,
- distribution grid that distributes power from the transmission grid to consumers,
- consumers, such as households, offices and industrial plants, and
- substations that convert voltage and serve as nodes, consumers can connect to.

Typical schematic of a power grid is given in figure 1. Usually large scale power grids transmit power using alternating current (AC), because voltage can easily be converted by using transformers. This is important to minimise transmission losses. The grid is usually partitioned by voltage levels

High voltage 110 kV to 330 kV,

Medium voltage 35 kV to 65 kV,

Low voltage 0.2 kV to 10 kV.

In every power grid the load balance of production and consumption must hold at all times. Since the AC grid is a network of coupled oscillators, in addition the synchronisation is a crucial aspect [2]. The fact that each power plant has different characteristics makes it even harder. For this reason, also DC grids are of interest, since there is no synchronisation problem.

For the distribution grid part, *microgrids* are becoming more and more relevant with the idea of prosumers (Nodes that can both consume and produce power. Unlike in a conventional power grid where the nodes can only exclusively be energy producers or consumers.) and reliability [2]. Reliability would be enhanced by having an "energy community" wihtin one microgrid, meaning that there is sufficient storage and production capacity for when the main grid connection is lost. Microgrids are also relevant in transitioning to renewable energy sources which allow microproduction (prosumers).

This is also interesting from the economic and accounting perspective. All consumers have to pay a fee proportional to the amount of energy used from the grid to the grid company, since the electricity transmission introduces losses and the grid needs to be maintained and repaired in case of accidents. Now, given that inside the microgrid there is sufficient amount of storage and production capacity, there could be energy market within the microgrid or between neighbouring microgrids. This means that the grid cost would decrease significantly if instead of 250 km the transmission distance is 1 km.

Right now, the three phase AC grid does not encourage efficient power directing. There is a study [3] done on a prototypical house which has solar panels and an electric car charger. The results show that while there is plenty of solar production, the owner still pays the full grid fee for both charging a car and inserted solar energy. This is because solar panels and car charger are connected to different AC phases. The author claims that it is possible to distribute energy inside the house first and then consume/insert the remaining energy, but it needs more advanced power electronics. This is a good example of how DC grids would be beneficial, as the power can be directed only by introducing a slight potential difference and without the need of three phases.

2 Power grids as complex networks

Power grids can be thought of as complex networks where production plants, substations and consumers are the nodes $v_i \in \mathcal{V}$ and the power lines are the edges $e_{ij} \in \mathcal{E}$. The nodes can have properties such as the maximum capacity and power lines are weighted in many studies by their impedance, which gives an idea of overall power loss [1].

Many statistical studies have been done and an overview can be found in [1]. These include real power grid networks from all over the world and investigate the degree distribution, betweenness distribution, average path length and fault tolerance.

Many of the power networks in Europe have been found to satisfy the small-world phenomena [1], where the average path length is small but has high clustering [4]. However, networks from other parts of the world do not necessarily support the claim and there is no consensus on what is the usual type of power grid networks [1]. These studies in [1] mostly include the transmission grid. The distribution grid deviates even more from the small-world network due to small clustering and is probably more closer to a scale-free network as there are a lot of consumers with only a few links and much less substations which act like hubs. Also, there is no typical degree distribution, in most cases an exponential distribution $f(k) \propto e^{-\alpha k}$ is found and others have found a power law $f(k) \propto k^{-\alpha}$ [1]. The main difference between these is that the exponential falls off much faster and the probability of having a highly connected node is very unlikely.

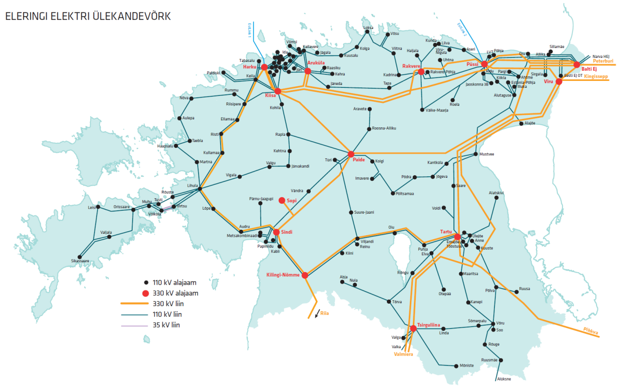

Figure 2: Estonian transmission grid. Figure is taken from [5].

Estonian transmission grid is shown in figure 2.

3 Microgrids in a military scenario

In the context of military the microgrids can be used for delivering power ondemand. Power is needed in the front line for flying drones, which are extensively used in the modern war for radio networks, video intelligence and organizing operations. In addition, field hospitals and control points need power for functioning. The problem is that in these areas there can be no heat or noise signature of a regular diesel generator and the battery logistics could attract too much attention. Microgrids allow to distribute power from a safer area. The most important features for a microgrid are

1. easy deployablity,

2. reliability,
3. resilience to attacks, some nodes or links might be destroyed,
4. masking in nature to stay unnoticed.

The idea in CAFA Tech is to construct a High Voltage Direct Current (HVDC) power grid. In a typical scenario there would be diesel generators 3 km to 10 km away from the destination, possibly distributed for redundancy. From the generators HVDC power lines are taken to subnodes which convert and distribute power. High voltage is used in order to reduce the mass of cables dragged on the field, since this is mostly a man powered operation. Higher voltage allows for using lower current which reduces the necessary cross section of the conductor. Direct current has a few advantages in this case.

1. The voltage converters can be small and light as high frequency for conversion can be used, thus reducing the required cross-section area of the transformer.
2. There is no synchronisation problem in the grid. The network can be rapidly expanded by hot-plugging new generators and nodes.
3. Many devices in military internally use DC, thus reducing the amount of converters and improving efficiency.

3.1 Simulation of the grid topology

I analyse the situation in a rectangular domain $D \subset \mathbb{R}^2$ which consists of 3 zones, generation, transmission and consumers, see figure 3. The nodes will also be in one of three categories $v_{g_i}, v_{d_j}, v_{c_k} \in \mathcal{V}$ respectively. The task is to find

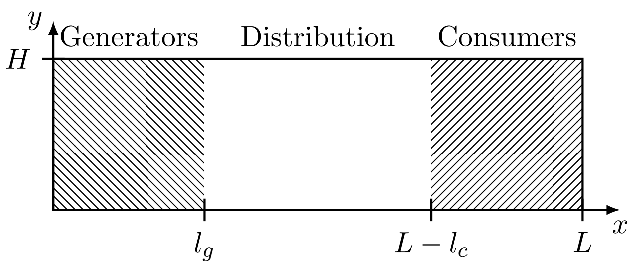

Figure 3: Simulation domain

best grid topology for distributing power. Best in this context means even distribution and fault tolerance under restricted amount of generators, wires and nodes. Also, the consumers should have the shortest possible paths to the producers to minimise electrical losses and amount of cable used. However, in a military scenario losses are not as important.

To distribute power evenly across consumer's area is very straight forward. One needs to evenly space the distribution nodes. For the optimisation process to connect all generator and consumer nodes, I assign each generator the maximum power it can generate $W_i = 1$, here I assume all generators are equal. Available power in a consumer node k can be measured by

$$W'(v_{c_k}, \mathcal{V}, \mathcal{E}) = \sum_{i=1}^{N_g} W_i[1 - \varepsilon\sigma(v_{c_k}, v_{g_i})], \quad (1)$$

where $\sigma(v_{c_k}, v_{g_i})$ is the path length between the nodes. In case there is no path, I take $\sigma = 1/\varepsilon$. Weighting generators by path length has a few reasons

- minimise losses, although this is not the most crucial factor in the military case,
- reduce the distribution network size, each node has a cost of setting up and maintenance.

In this simulation I keep $\varepsilon \ll 1$ small.

3.1.1 Fault tolerance

A network is said to be fault tolerant when some fraction of links or nodes are removed but the grid still has a giant component of connected nodes. One way for analysing network error tolerance is to do numerical experiments, where networks with different topologies are generated and then the nodes or edges are removed. Then the average path length l and size S of the largest cluster are measured as a function of fraction of nodes removed f. Fraction of removed nodes is a critical fraction f_c if the giant component size $S < S_c$, where $S_c \ll 1$ is a parameter. The removal can be either random or targeted. [6]

Fault tolerance has been heavily studied for standard graph topologies (small world, scale-free, random, etc.) in case of both *random* and *targeted* attacks, see for example [6]. In the military scenario, we have to consider something in between random and targeted attacks, since it is unlikely for the enemy to uncover the full network to make targeted attacks. But it is likely that they discover a few nodes and decide to remove the one with the highest degree. Also, there is the spacial distribution of nodes. Here, I assume that the probability of a node to be removed is proportional to distance from the dangerous region, so one could choose

$$p_{removal}(i) \propto x \tag{2}$$

In this simulation I remove the nodes as follows. Consumer nodes are removed with some probability p_c. This represents the dynamics of the troops. If some troop decides to move to other place, we do not want to rewire the network. Generators are removed with a smaller probability p_g and distribution nodes with a similar probability p_d to consumer nodes.

For the fault tolerance, we can not just use the size S of the giant component as the criterion because the importance is of the connection between generator and consumer nodes. Thus, I propose to take the average available power

$$\langle W' \rangle = \sum_{k=1}^{N_c} \frac{W'(v_{c_k})}{N_c}. \tag{3}$$

The critical fraction f_c is then the fraction of nodes to be removed in order for $\langle W' \rangle < W'_c$. For example, if $W'_c = 1/3$ then an average consumer is connected to only one generator that is 3 links away.

3.1.2 Simulation

The simulation assumes there are restricted amount of generators N_g, consumers N_c, nodes N_d and wires N_e. The objective is to maximize

$$g(\mathcal{V}, \mathcal{E}) = \lambda_1 f_c + \lambda_2 \frac{1}{W_t(1-\varepsilon)} \langle W' \rangle \tag{4}$$

where W_t is total available power in the grid and f_c is calculated as described by (3). The weights $\sum \lambda_i = 1$ are used for optimising different goals. The objective function is thus between 0 and 1. This ensures that generators are connected to consumers by shortest paths, but also that by removing nodes, power is still delivered.

I use Metropolis-Hastings algorithm for the topology optimization since the objective function (4) is a function of the whole graph and edge information has no continuous variables to optimise for. The algorithm steps are described below.

1. All nodes will be randomly distributed across the area. Edges are added according to Erdös-Renyi random graph model.
2. Choose a random edge $e = (i, j) \in \mathcal{E}$.
3. Rewire the edge to $e' = (k, l)$, where k and l are new random nodes. Construct a new graph with edges $\mathcal{E}'$.
4. Calculate acceptance ratio
$$\Delta g = g(\mathcal{V}, \mathcal{E}') - g(\mathcal{V}, \mathcal{E})$$
$$\alpha = \min\left\{1,\ e^{\Delta g/T}\right\},$$
where $T = \beta^t$ is the temperature for time step t and $0 < \beta < 1$ the cooling parameter.
5. Generate a random sample $u \sim U(0, 1)$.
6. If $u \leq \alpha$ then we accept the new position and set $e = e'$.
If $u > \alpha$ then we reject the move and won't rewire it for the next step.
7. Repeat steps 2-6 for M steps.

3.1.3 Topology optimisation results

At first I try with a small graph so that it is easy to draw some qualitative results. The probability of removal is uniform across the grid. Only nodes with degree of 1 or more are removed.

I used the following parameters

$$\begin{aligned} L &= 1, & H &= 0.5, & N_e &= 13, \\ N_g &= 3, & N_c &= 5, & N_d &= 5, \\ W_i &= 1, & \varepsilon &= 0.05, & \beta &= 0.996. \end{aligned}$$

The initial configuration of the simulation is in figure 4. The algortihm is run for 2000 time steps and the best result is in figure 5.

The results with different initial configurations are qualitatively similar. Distribution nodes have no significant role and generators rather have a straight connection with the consumers. Leftover edges are used to form a tighter network among either generators or consumers. The objective value of the Markov chain is on figure 6. We can see that the system explores the space in the first 1000 steps and then nicely finds the maximum configuration.

The conclusion is that each consumer needs to connect to as many generator nodes as possible. The leftover wires should be used to form a tighter network between either consumers or generators.

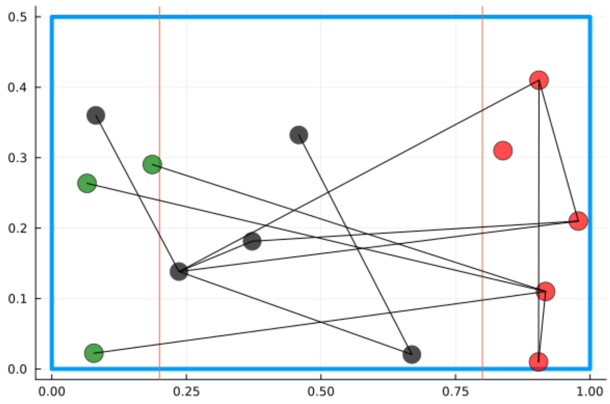

Figure 4: Initial configuration of the simulation. *Green* nodes represent generators, *black* are distribution nodes and *red* are the consumer nodes.

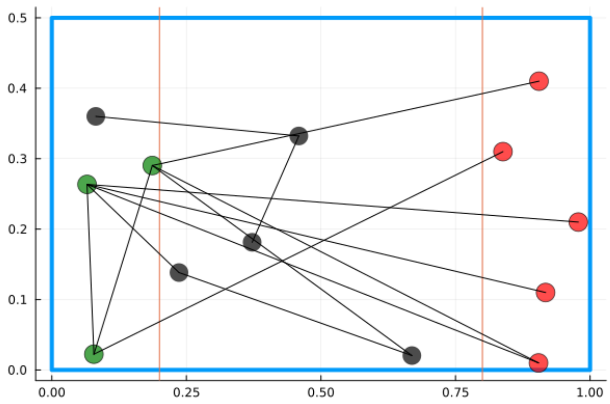

Figure 5: Best result of the edge optimisation, with $\lambda_1 = \lambda_2 = 1/2$ and $\lambda_3 = 0$.

This way, maximum power reaches every consumer node and the power is still transmitted in case of attacks to nodes or dynamics of the troops.

4 Conclusions

Further, the problem could be analysed with a more complicated removal probability field with choosing a few nodes from the network and removing the one with the highest degree. The removal probability field could be modified to represent the landscape, danger zones, etc. Thus, make the problem space embedded and optimise for best node positions as well. In addition, one could add edge attacks. Since it is more likely for the enemy to find a wire than a node.

One problem I encountered many times is that, the system tries to completely eliminate all distribution nodes from the network. I added the

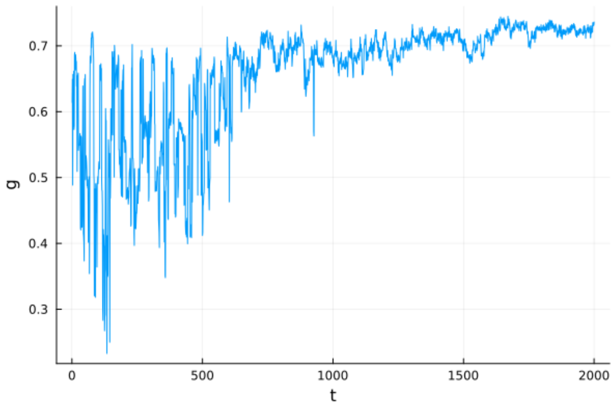

Figure 6: Objective value as a function of time step.

condition of not removing a node that has fewer than 2 connections, these are basically removed and this produces artificially high critical fraction values. This way the system optimises for the distribution nodes to be "end nodes" of the power network which have no significant effect on the topology when removed. I could not find a deterministic way of detecting such unimportant nodes. Centrality measures are continuous variables and need a critical value for determining if a node is important enough. How could one find that critical value? The conclusion from this is that distribution nodes add too much complexity and should be avoided in a real scenario. However, the model should include maximum length of a wire that one soldier can carry.

Additionally, the model should optimise for the network size. The consumer and generator nodes are fixed but the distribution nodes and amount of wires determines the topology. In the case described here a fully connected network would be the best case, but one should find a balancing property. Each wire has a set up cost, so for example, one could also minimise the total wire length.

References

[1] G. A. Pagani and M. Aiello. *The Power Grid as a Complex Network: A Survey*. 2012. DOI: 10.48550/arXiv.1105.3338. Pre-published.

[2] P. H. Nardelli et al. "Models for the Modern Power Grid". In: *The European Physical Journal Special Topics* 223.12 (2014), pp. 2423–2437. DOI: 10.1140/epjst/e2014-02219-6.

[3] A. Blinov. “Challenges of ZEB and Full-Electric Lifestyle”. In: The 5th Estonian DC Innovation Workshop. TalTech, 2024.

[4] D. J. Watts and S. H. Strogatz. “Collective Dynamics of ‘Small-World’ Networks”. In: *Nature* 393.6684 (1998), pp. 440–442. DOI: 10.1038/30918.

[5] *Elektri ülekandevõrk — Elering*. URL: https://elering.ee/elektri-ulekandevork (visited on 04/20/2025).

[6] R. Albert and A.-L. Barabasi. “Statistical Mechanics of Complex Networks”. In: *Reviews of Modern Physics* 74.1 (2002), pp. 47–97. DOI: 10.1103/RevModPhys.74.47.